

Measurement Dependence Inducing Latent Causal Models



universität
wien

Alex Markham and Moritz Grosse-Wentrup

Research Group Neuroinformatics, University of Vienna

contact: alexander.markham@univie.ac.at

website: <https://causal.dev/>

Abstract

We consider the task of causal structure learning over a set of measurement variables with no direct causal relations and whose dependencies are induced by unobserved latent variables. We call this the *measurement dependence inducing latent* (MeDIL) Causal Model, or MCM, framework. We show that this task can be framed in terms of the graph theoretical problem of finding edge clique covers, resulting in a simple algorithm for returning minimal MeDIL causal models (minMCMs). This algorithm is non-parametric, requiring no assumptions about linearity or Gaussianity. Furthermore, despite these rather weak and general

assumptions, we are able to show that *minimality* in minMCMs implies three rather specific and interesting properties: first, minMCMs lower bound (i) the number of latent causal variables and (ii) the number of functional causal relations that are required to model a complex system at *any* level of granularity; second, a minMCM contains no causal links between the latent variables; and third, in contrast to factor analysis, a minMCM may require more latent than measurement variables.

The Edge Clique Cover (ECC) Problem

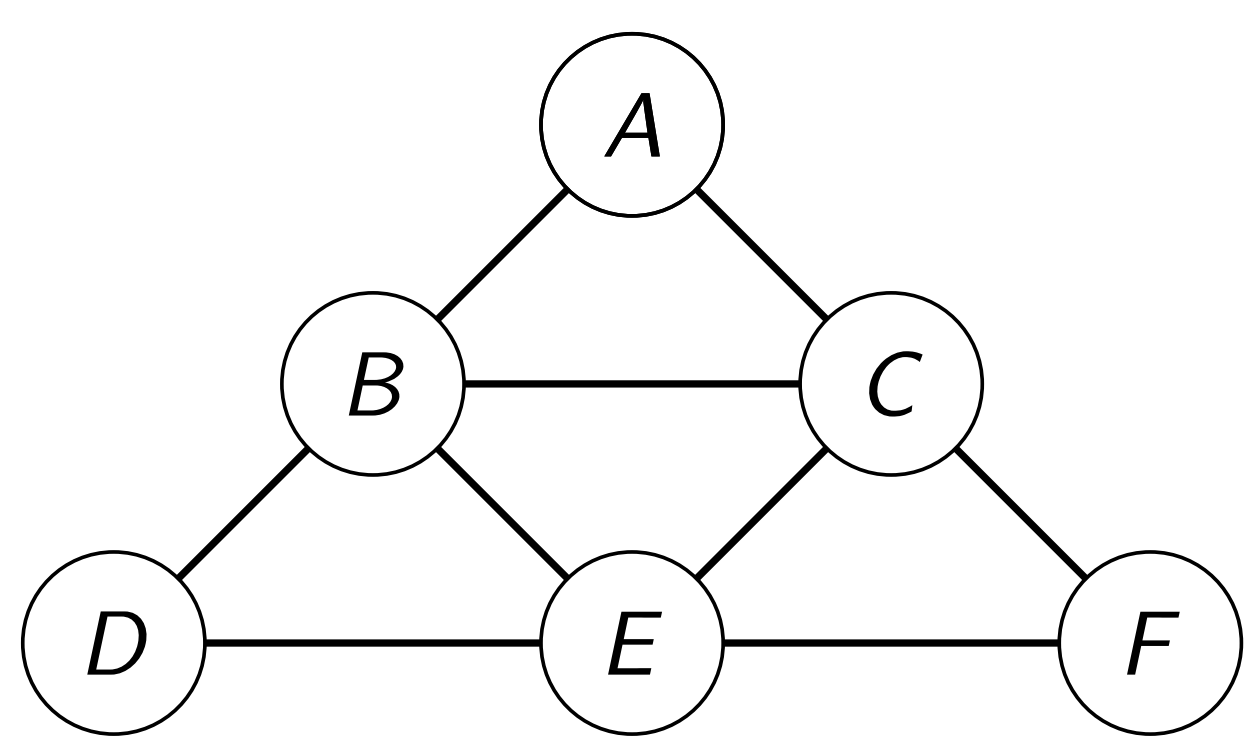


Figure 1: example undirected graph

The set of all maximal cliques for the above graph (i.e., the set consisting of all four 3-graphs) is an *edge clique cover* (ECC). Notice, however, that the clique formed by nodes B , C , and E can be removed and the three remaining cliques still cover all edges. These three cliques together form a *minimal ECC*. Finding a minimal ECC is more complicated and has higher complexity than finding the set of all maximal cliques.

Furthermore, there are two different kinds of minimality, clique-minimal and assignment-minimal:

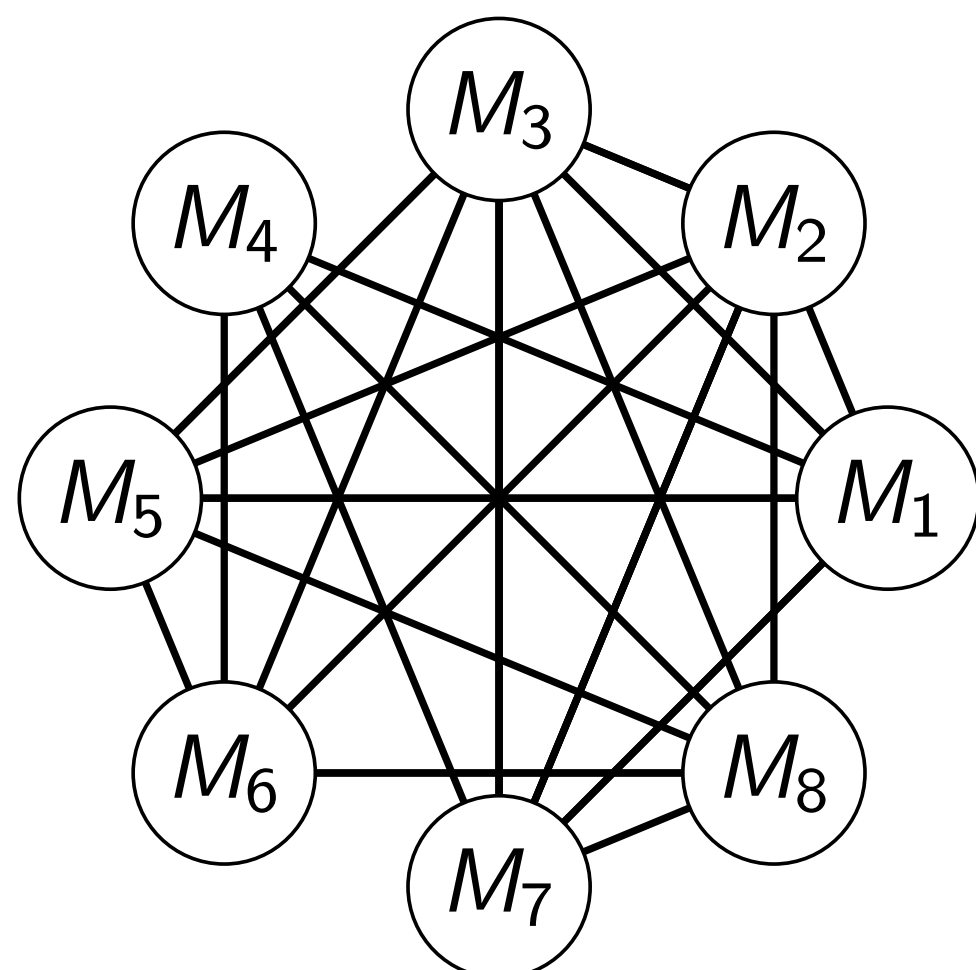


Figure 2(a): example undirected graph $D(\mathbf{M})$ over variables $\mathbf{M} = \{M_1, \dots, M_8\}$

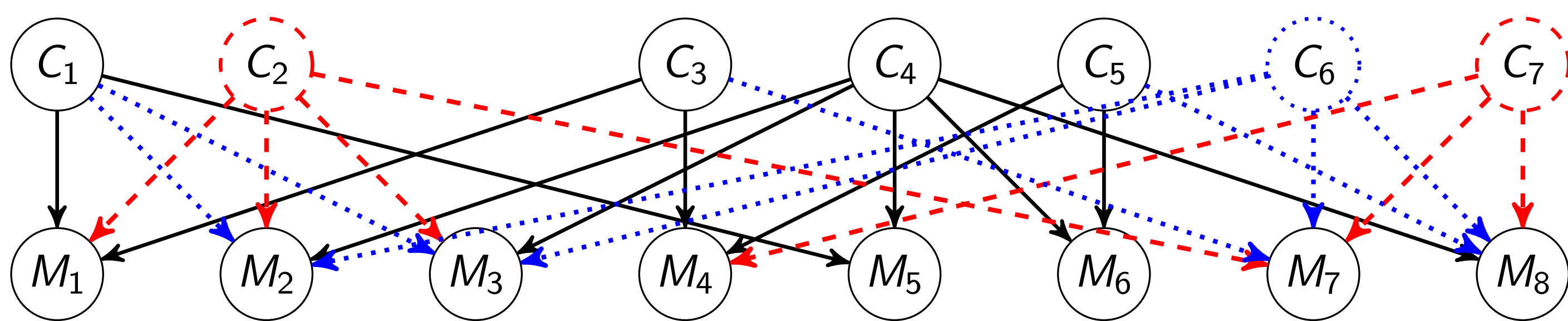


Figure 2(b): each C_i corresponds to a maximal clique in $D(\mathbf{M})$ and each directed edge represents assignment—dashed red edges/vertices are redundant for clique-minimality while blue dotted edges/vertices are redundant for assignment-minimality;

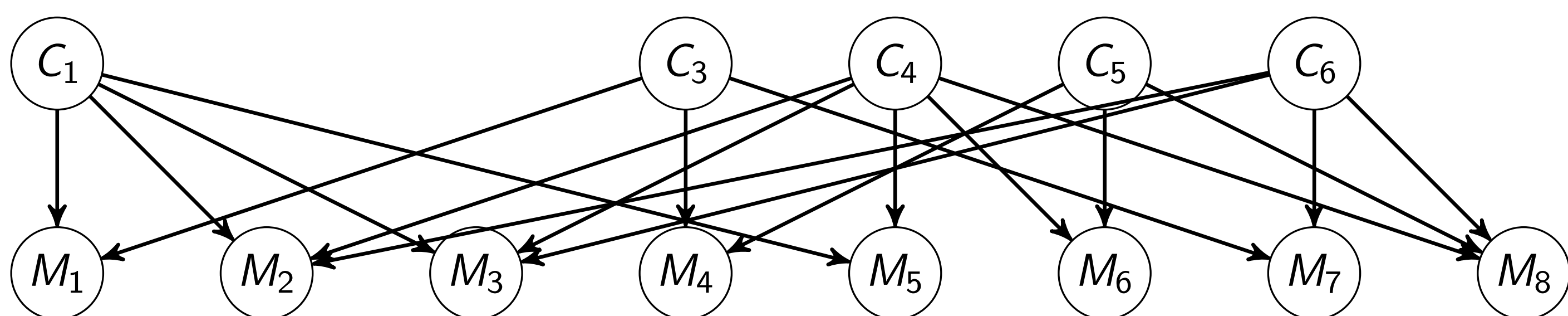


Figure 2(c): 5 cliques C_i and 19 assignments for clique-minimal ECC over $D(\mathbf{M})$

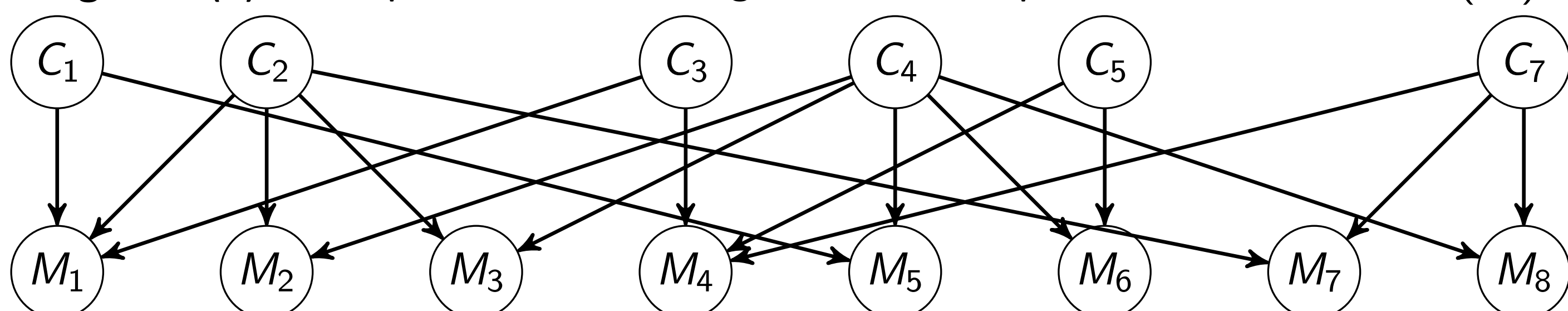


Figure 2(d): 6 cliques C_i and 18 assignments for assignment-minimal ECC over $D(\mathbf{M})$

MeDIL Causal Models

Definition 1 (Measurement Dependence Inducing Latent Causal Model (MCM)). A graphical MCM is a DAG, given by the triple $\mathcal{G} = (\mathbf{L}, \mathbf{M}, \mathbf{E})$. \mathbf{L} and \mathbf{M} are disjoint sets of vertices, while \mathbf{E} is a set of directed edges between these vertices, subject to the following constraints:

1. all vertices in \mathbf{M} have in-degree of at least 1 and out-degree of 0
2. all vertices in \mathbf{L} have out-degree of at least 1
3. \mathbf{E} contains no cycles

To learn a minMCM for a given distribution of measurement variables, we represent the measurement variables \mathbf{M} as an *undirected dependency graph* upon which ECC finding algorithms can be applied. We denote this graph $D(\mathbf{M})$, and construct it by putting an undirected edge between two measurement variables if and only if they are unconditionally dependent. These dependencies can be learned from a set of samples via permutation-based hypothesis testing using non-linear measures of dependence, such as the Hilbert-Schmidt Independence Criterion (HSIC) or the distance correlation.

Algorithm 1: constructing a minimal MeDIL causal model (minMCM)

Input : $D(\mathbf{M})$ over the measurement variables \mathbf{M}

Output: vertex-minimal or assignment-minimal MCM \mathcal{G} over \mathbf{M}

- 1 initialize edgeless graph with a vertex for each $M \in \mathbf{M}$;
- 2 use `find_cm` or `find_am` to get an edge clique cover of $D(\mathbf{M})$;
- 3 **for each clique C in the cover do**
- 4 | add vertex L with edges directed to each $M \in C$;
- 5 **end**

Some interesting properties include:

- minMCMs lower bound (i) the number of latent causal variables and (ii) the number of functional causal relations that are required to induce the measurement variables
- minMCM has property that for all measurement variables, conditional independence relations are implied by unconditional independence relations (this is analogous to the global Markov property of a Markov random field)
- a minMCM contains no causal links between the latent variables
- in contrast to factor analysis or independent component analysis, a minMCM may require more latent than measurement variables.

Conclusion

- some (in)dependence structures cannot be represented by a DAG over the corresponding variables but *can* be represented by an undirected dependency graph
- these structures are common for *measurement* variables, which are noisy copies or combinations of unobserved latents, e.g., in fMRI, calcium imaging, and psychiatric or econometric questionnaire data
- we propose the MeDIL causal model framework for use on such data and provide an algorithm for finding a minimal MCM for a given distribution of measurement variables.